

Chapter 1

Introduction to Linear Algebra

1.1 Systems of Two Equations

Exercises

1. Solve the following systems of linear equations.

(a)
$$\begin{aligned}x + y &= 2 \\ 3x - 2y &= -1\end{aligned}$$

(b)
$$\begin{aligned}3x + 2y &= 5 \\ 3x - 2y &= 7\end{aligned}$$

(c)
$$\begin{aligned}12x - 7y &= 17 \\ 5x - 3y &= 7\end{aligned}$$

(d)
$$\begin{aligned}25x + 62y &= 27500 \\ 28x + 42y &= 26900\end{aligned}$$

2. The demand and cost for a certain product are given by the following equations

(a) $x = -2p + 4000$, $C = 2000 + 5x$;

(b) $x = -p + 90$, $C = 500 + 7x$.

Find the price for which break-even point is reached. (Suppose that the demand equals the supply.)

3. The dependence between the quantity x of a certain good that is sold in a market and the price p of a unit of the good is called *demand function*, and the dependency between the quantity x of the good that is produced and offered in the market and the price p is called *supply function*.

A market is called *balanced* if the supply equals the demand.

Find the quantity and the price which give a balanced market for a particular good, if the demand function is given by $x = -5p + 10$ and the supply function is given by $x = 2p - 7.5$.

4. The demand function is given by $x = -\frac{p^2}{5} + 10$ and the supply function is given by $x = \frac{p^2}{2} - 7.5$.

(a) Determine the price of the good which gives a balanced market.

(b) Find the quantity of the good which gives a balanced market.

5. A manufacturer estimates that the fixed cost of his business will be 2,500 € per month and that the variable cost will increase linearly. If the variable cost for producing 1500 units will be 22,500 € per month, and the selling price under the conditions of balanced market is 25 € per unit, find the quantity of the product for which his business will reach the break-even point. What is the value of total cost at that point?
6. Find the x and y intercepts for the line $3x + 2y = 12$, and graph the line.
7. Find the slope of the line $x + 2y = 4$.
8. Find the equation of the line through the points $(3, 2)$ and $(-1, 0)$.

9. The produce company AppleMax sells apples for 20 € per box for the first 10 boxes. Orders for more than 10 boxes receive a discount of 15% on the boxes purchased beyond 10. Find an expression for the cost of an order of x boxes, graph that cost versus x , and use that expression to find the cost of an order of 18 boxes.
10. Murphy's Muffin Shoppe makes two sizes of raisin muffins using prepackaged dough and raisins. Each large muffin uses 15 decagrams of dough (1 decagram is 10 grams) and 6 decagrams of raisins, and each small muffin uses 6 decagrams of dough and 3 decagrams of raisins. Each day the shop receives 1350 decagrams of dough and 600 decagrams of raisins. How many large muffins and small muffins should be baked each day to use up all the dough and all the raisins?