

## 2.5 Loans and Amortization

### Exercises

1. A loan of 50,000 €, with interest at a rate of 5% is amortized over 12 years. What is the size of a periodic payment, if the payments and compounding are
  - (a) semiannual;
  - (b) quarterly;
  - (c) monthly?
2. A home loan is made for 150,000 € at 9% annual interest, compounded monthly, for 30 years. What is the monthly mortgage payment on this loan?
3. Determine the monthly car payment for a new car costing 15,675 €, if there is a down payment of 4,000 € and the car is financed over a 5 year period at 6%, compounded monthly.
4. Show that if a loan of  $K$  € at 9%, compounded monthly, is amortized over  $n$  years then the size of each payment is

$$R = \frac{Pi}{1 - (1 + i)^{-12n}},$$

where  $i = \frac{p}{100 \cdot 12}$  is the monthly interest rate (expressed as a decimal).

5. What loan can be amortized in 10 years, at an interest rate of 6%, by periodic payments of 1,000 € if the payments and compounding are
  - (a) annual;
  - (b) semiannual;
  - (c) quarterly?
6. Suppose a family figures it can handle monthly mortgage payments of no more than 1,200 €. What is the largest amount of money they can borrow, assuming the lender is willing to amortize over 30 years at 9%, compounded monthly?

7. How many annual payments of size 1,000 € are needed to amortize a loan of 14,877.47 €, at 3%, compounded annually?
8. How long (in years) does it take to amortize a loan of 85,000 €, at 2.4%, compounded quarterly, by quarterly payments of 5,089.21 €?
9. How many semiannual payments of 8,015.51 € are needed to amortize a loan of 72,000 € at 4%, compounded semiannually?
10. How many annual payments of 1,000 € are needed to amortize a loan of 14,877.47 € at 3%, compounded annually?
11. A debt of 200,000 €, at 8%, compounded annually, is amortized over 5 years by equal annual payments. Determine:
  - (a) the size of a payment;
  - (b) the balance reduction from the first payment;
  - (c) the balance reduction from the fourth payment.
12. A loan of 100,000 € with interest at 8%, compounded monthly, is amortized over 15 years by equal monthly payments. Determine the balance reduction from the last payment.
13. Show that if a debt of  $P$  € with interest at an annual rate of  $p\%$ , compounded annually, is amortized over  $n$  years, then the balance reduction from the first annual payment is

$$B_1 = P \frac{r - 1}{r^n - 1}.$$

14. Show that if a debt with interest at an annual rate of  $p\%$ , compounded annually, is amortized over  $n$  years by annual payments of  $R$  €, then
  - (a) the balance reduction from the first payment is

$$B_1 = Rr^{-n},$$

- (b) the balance reduction from the  $i$ -th payment is

$$B_i = Rr^{i-n-1}.$$

15. A debt of 10,000 € with interest at 12%, compounded monthly, is amortized over 1 year by equal monthly payments. Develop the amortization schedule.
16. A debt of 50,000 €, at 9%, compounded quarterly, is amortized over 3 year by quarterly payments. Develop the amortization schedule.
17. A loan of 500,000 €, with interest at 7%, compounded annually, is amortized over 10 year by annual payments. Determine the unpaid balance after 7 payments have been made.
18. A debt of 1,000,000 € at 6%, compounded monthly, is amortized over 15 year by monthly payments. Determine the unpaid balance after 120 payments have been made.
19. Show that if loan of  $P$  € with interest at  $p\%$ , compounded annually, is amortized over  $n$  years, then the unpaid balance  $U_i$  after  $i$  payments have been made is

$$U_i = P \frac{r^n - r^i}{r^n - 1}.$$

20. Express the formula from Exercise 19 if the frequency of payments (and compounding periods) is  $m$  per year.